Revision Equivalence for Ranking Functions

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Motivation: Total Preorders

Total Preorders (TPOs) over possible worlds are widely used in *non-monotonic reasoning* (NMR) and *belief revision* (BR).

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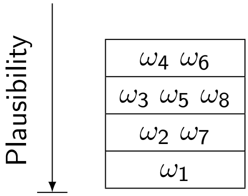

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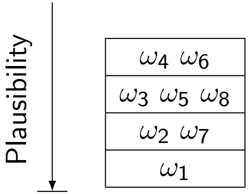

NMR

inference relations based on TPOs guarantee KLM postulates

mentioned by Lucas :)

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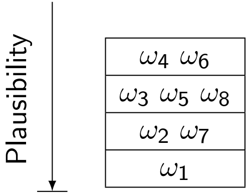

NMR

inference relations based on TPOs guarantee KLM postulates

mentioned by Lucas :)

BR

revision operators based on TPOs satisfy AGM & DP postulates

mentioned by Clayton :)

Motivation: Conditionals

Total preorders over possible worlds essentially encode conditional information.

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Ω	A	B	C	...
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ω_2	0	0	1	
ω_3	0	1	0	
ω_4	0	1	1	
ω_5	1	0	0	
ω_6	1	0	1	
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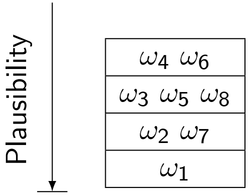
Total Preorder

over Ω



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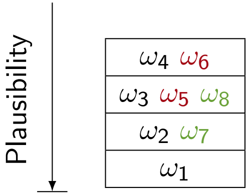

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Models of $A \wedge \neg B$

Models of $\neg A$

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Total Preorder
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Plausibility

ω_4 ω_6
ω_3 ω_5 ω_8
ω_2 ω_7
ω_1

Models of $A \wedge B$

Models of $A \wedge \neg B$

Models of $\neg A$

$\implies$ *Conditional*: $(B|A)$ meaning "If A, then (usually/plausibly) B".
(since $A \wedge B$ is more plausible than $A \wedge \neg B$)

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Ordinal Conditional Function (OCF) over Ω:

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Open Questions:

- Which ranking function would be *most adequate* to represent the total preorder?
- And when *does it matter* which ranking representation we choose?

Motivation: Empty Layers during Revision

Empty (in-between) layers in OCFs show their power during *belief change* operations.

5	
4	
3	$\omega_7^{\neg A}$ ω_8^{A}
2	ω_4^{A} $\omega_5^{\neg A}$ ω_6^{A}
1	ω_2^{A} $\omega_3^{\neg A}$
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vs.

5	
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Scenario: Revision by A

$\Rightarrow$ Models of A become more plausible (rank -1).

$\Rightarrow$ Models of $\neg A$ become less plausible (rank $+1$).

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$\xrightarrow{*A}$

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Revision Equivalence

Idea: $\kappa_1 * A \cong \kappa_2 * A$ for all A.

$\Rightarrow$ **Representation Invariance**

Overview

Formal Basics

Propositional Logic

- Σ : signature containing atomic propositions, i.e. $\Sigma = \{a, b, c, \dots\}$.
- $\mathcal{L}$: propositional language over signature Σ.
- Ω : set of propositional interpretations (*possible worlds*) over Σ.
- $\omega \models A$ iff $A \in \mathcal{L}$ holds in $\omega \in \Omega$.
- $Mod(A) := \{\omega \in \Omega \mid \omega \models A\}$.

Conditionals

- Language: $(\mathcal{L}|\mathcal{L}) := \{(B|A) \mid A, B \in \mathcal{L}\}$
- $(B|A)$ formalizes: "If A, then *usually* B."

Epistemic States

Epistemic States [Darwiche & Pearl, 1997]

In the Darwiche-Pearl Framework of iterated revision, an epistemic state Ψ is represented by a *total preorder* (TPO) $\preceq_\Psi$ over a set of possible worlds Ω.

- $\omega_1 \preceq_\Psi \omega_2$ iff the possible world $\omega_1 \in \Omega$ is at least as plausible as $\omega_2 \in \Omega$ in Ψ.
- $\Psi \models (B|A)$ iff at least one possible world satisfying $A \wedge B$ is more plausible than all worlds satisfying $A \wedge \neg B$.

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Properties of Total Preorders

Technically, $\preceq_\Psi$ is a relation $\preceq_\Psi \subseteq \Omega \times \Omega$ with the following properties:

1. $\omega_1 \preceq_\Psi \omega_2$ or $\omega_2 \preceq_\Psi \omega_1$ holds. (Totality)
2. $\omega_1 \preceq_\Psi \omega_2$ and $\omega_2 \preceq_\Psi \omega_3$ imply $\omega_1 \preceq_\Psi \omega_3$. (Transitivity)

for all $\omega_1, \omega_2, \omega_3 \in \Omega$.

DP-style Iterated Revision

Proposition [Darwiche & Pearl, 1997]

A revision operator $*$ that assigns a posterior epistemic state $\Psi * A$ to a prior state Ψ and a proposition A is an *AGM revision operator for epistemic states* iff there exists a *total preorder* (TPO) $\preceq_\Psi$ on Ω with $Mod(Bel(\Psi)) = \min(\Omega, \preceq_\Psi)$ such that

$$Mod(Bel(\Psi * C)) = \min(Mod(C), \preceq_\Psi)$$

holds for every proposition C.

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$\omega_1^{\neg A}$

$\longleftarrow$ ω_2: most plausible world after revision by A

$\longleftarrow$ ω_1: most plausible world (currently)

Example Revision Operators for TPOs

Natural Revision [Boutilier, 1993]

The *natural revision operator* $\bullet_n$ is defined by the following condition:

$$\omega \preceq_{\Psi \bullet_n A} \omega' \quad \text{iff} \quad \begin{array}{l} (1)\ \omega \in \min(Mod(A), \preceq_\Psi), \text{ or} \\ (2)\ \omega, \omega' \notin \min(Mod(A), \preceq_\Psi) \text{ and } \omega \preceq_\Psi \omega' . \end{array}$$

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$\longrightarrow$ "pull down" all models of A (and keep relative order within models/non-models)

Ranking Functions

Definition: Ordinal Conditional Function [Spohn, 1988]

An *ordinal conditional function* (OCF) or *ranking function* is a mapping

$$\kappa : \Omega \to \mathbb{N} \cup \{\infty\}$$

with $\kappa^{-1}(0) \neq \emptyset$.

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The rank of a formula A is given by $\kappa(A) = \min\{\kappa(\omega) \mid \omega \models A\}$.

The acceptance relation w.r.t. conditionals is defined by

$$\kappa \models (B|A) \quad \text{iff} \quad \kappa(A \wedge B) < \kappa(A \wedge \neg B)\,.$$

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Inferential Equivalence $\kappa \cong \kappa'$: $\kappa(\omega_1) \leq \kappa(\omega_2)$ iff $\kappa'(\omega_1) \leq \kappa'(\omega_2)$ for all $\omega_1, \omega_2 \in \Omega$.

Strategic c-Revisions for OCFs

c-Revisions for OCFs [Kern-Isberner, 2004]

Let κ be an OCF and $\Delta = \{(B_1|A_1), \ldots, (B_n|A_n)\}$ a set of conditionals. Then a *c-revision of* κ *by* Δ is an OCF $\kappa^* = \kappa * \Delta$ of the form

$$\kappa^*(\omega) = \kappa_0 + \kappa(\omega) + \sum_{\substack{1 \leqslant i \leqslant n \\ \omega \models A_i \wedge \neg B_i}} \eta_i$$

with nonnegative integers $\eta_i \in \mathbb{N}$ for each $(B_i|A_i)$ ensuring that $\kappa^* \models \Delta$,

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with nonnegative integers $\eta_i \in \mathbb{N}$ for each $(B_i|A_i)$ ensuring that $\kappa^* \models \Delta$,

$\longrightarrow$ The rank of a world ω is shifted by a sum of "penalties" for falsifying conditionals.

Strategic c-Revisions for OCFs

c-Revisions for OCFs [Kern-Isberner, 2004]

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Selection Strategies [Kern-Isberner, Sezgin, Beierle, 2022]

A *selection strategy* is a mapping $\sigma : (\kappa, \Delta) \mapsto \vec{\eta}$ where $\vec{\eta}$ is a solution to the constraints above.

Transformations between TPOs and OCFs

[Kern-Isberner, Sezgin, Beierle, 2022]

$$\begin{tikzcd}[column sep=huge] \begin{matrix}\boldsymbol{\Psi}\\ \text{Total Preorders}\end{matrix} \arrow[r, bend right, "\textbf{generalize}"'] & \begin{matrix}\boldsymbol{\kappa}\\ \text{Ranking Functions}\end{matrix} \arrow[l, bend right, "\textbf{implement}"'] \end{tikzcd}$$

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implement

$\tau : \kappa \mapsto \Psi_\kappa$

$\omega_1 \preceq_\kappa \omega_2$ iff $\kappa(\omega_1) \leqslant \kappa(\omega_2)$

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Total Preorders

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Note: $\tau \circ \rho = id$, but $\rho \circ \tau \neq id$ in general.

Example: Transformations

Let a ranking function κ be defined as:

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Now $\rho(\Psi_\kappa)$ returns κ_{Ψ_κ}:

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Motivation: Revision Equivalence

5	
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We want to preserve equivalence of OCFs during revision.

Revision Equivalence

Let $*$ be an iterated revision operator for OCFs, taking (sets of) propositions resp. conditionals as input.

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Some Negative Results

Proposition

Let κ_1, κ_2 be two different, but (inferentially) equivalent ranking functions which both have at least two layers such that their lowermost layer Ω_0 has more than one element. Then there is a strategic c-revision operator $*_\sigma$ and $A \in \mathcal{L}$ such that $\kappa_1 *_\sigma A \not\cong \kappa_2 *_\sigma A$.

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Theorem

Let κ_1, κ_2 be two different ranking functions. Then κ_1, κ_2 are conditionally revision equivalent with respect to strategic c-revisions complying with (Stability) iff both κ_1, κ_2 have exactly two layers Ω_0, Ω_1 such that $\Omega_0 = \{\omega_0\}$ contains exactly one element.

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$\Longrightarrow$ Revision equivalence is not easy to achieve in general.

But we are not done yet!

How can we achieve Revision Equivalence?

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Approach 1: Refine our Notion of Revision Equivalence

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- Specific strategies for c-Revisions: $\sigma(r \cdot \kappa, \Delta) = r \cdot \sigma(\kappa, \Delta)$

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Approach 2: Use TPO Revision Operators and Transformation Functions

When revising κ by φ, essentially “mimic” the behavior of a TPO revision operator. Two options:

- Use a TPO revision operator $\bullet$ directly: $\rho(\tau(\kappa) \bullet \varphi)$.
- Construct an OCF revision operator $*$ such that: $\tau(\kappa * \varphi) \;=\; \Psi \bullet \varphi$.

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Definition: Linear Equivalence

Two OCFs κ_1, κ_2 over Ω are called *linearly equivalent* if $\kappa_2 = q \cdot \kappa_1$ for some positive rational number q.

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Theorem (Strategies for Linear Revision Equivalence)

If $\kappa_2 = q \cdot \kappa_1$, and σ is a selection strategy that satisfies

$$\sigma(r \cdot \kappa, \Delta) = r \cdot \sigma(\kappa, \Delta), \qquad (\text{Mult}^c)$$

then $\kappa_2 *_\sigma \Delta = q \cdot (\kappa_1 *_\sigma \Delta)$ for any (consistent) set $\Delta = \{(B_1|A_1), \ldots, (B_n|A_n)\}$.

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Hence for strategic c-revisions $*_\sigma$ where the strategy σ satisfies (Mult^c) ...

- linearly equivalent ranking functions are (univ.) conditionally revision equivalent.
- even the factor q is respected!

Preservation of Linear Equivalence

We can formalize this property of "respecting the factor" with the following definition.

Definition: Preservation of Linear Equivalence

A revision operator $*$ *preserves linear equivalence* if for any linearly equivalent κ_1, κ_2 such that $\kappa_2 = q \cdot \kappa_1$ and for any proper input φ, it holds that $\kappa_2 * \varphi = q \cdot (\kappa_1 * \varphi)$.

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From the theorem on the previous slide we can conclude:

Strategic c-revisions $*_\sigma$ where the strategy σ satisfies (Mult^c) preserve linear equivalence under revison by sets of conditionals.

Revision Equivalence via TPO Revisions (1/2)

In principle, every revision operator $\bullet$ for total preorders can be used to define a revision operator for ranking functions κ by utilizing the transformation functions τ and ρ.

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where φ is an appropriate input for $\bullet$.

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Proposition

Let κ be an OCF, let $\circledast$ be constructed from a revision operator $\bullet$ for total preorders as described in (1), and let φ be an appropriate input for $\bullet$. Then the OCF-revision operator $\otimes$ defined by

$$\kappa \otimes \varphi \; = \; (\kappa \circledast \varphi) \cdot \min\{\kappa(\omega) \mid \omega \in \Omega, \kappa(\omega) > 0\}$$

preserves linear equivalence.

Revision Equivalence via TPO Revisions (2/2)

OCF Revision $*$ "mimics" TPO Revision $\bullet$

We define $*$ accordingly from scratch such that

$$\tau(\kappa * \varphi) = \Psi \bullet \varphi \tag{2}$$

holds for every epistemic state Ψ properly represented by a total preorder, every $\kappa \in \tau^{-1}(\Psi)$, and every new information φ.

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Proposition

Let $*$ be a revision operator for ranking functions that satisfies (2) for some revision operator $\bullet$ for total preorders, and let φ be an appropriate input for $\bullet$. Let κ_1, κ_2 be equivalent OCFs. Then

$$\kappa_1 * \varphi \;\cong\; \kappa_2 * \varphi \,.$$

Example: OCF Versions of Elementary Operators

Natural OCF-Revision

$$(\kappa *_n A)(\omega) = \begin{cases} 0 & \text{iff } \omega \models A \text{ and } \kappa(\omega) = \kappa(A), \\ 1 + \kappa(\omega) & \text{otherwise.} \end{cases}$$

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Corollary

Let κ_1, κ_2 be equivalent OCFs. Then κ_1 and κ_2 are (propositionally) revision equivalent with respect to both $*_n$ and $*_\ell$.

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