Introducing Defeasibility into Standpoint Logics

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- Different perspectives disagree with each other.
- Viewpoints can believe in rules that have exceptions.

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Construct a logical system that can represent multiple agents with:

- **External** contradictions between standpoints.
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2. Describe syntax and semantics for **Defeasible Restricted Standpoint Logic** (DRSL).
3. Create a version of (non-monotonic) entailment to reason with this logic.
4. Show that this system of reasoning can be described with an algorithm and a semantic structure.

Part of a broader project of integrating defeasibile reasoning beyond the propositional case!
(Eg. Description Logics, Modal Logics, First-Order Logic)

KLM Defeasible Reasoning: Syntax

Propositional KLM Logic [5]

The **language** for KLM defeasible reasoning consists of adding a rational consequence operator (defeasible implication) between Boolean expressions of the form,

$$\alpha \mid\!\sim \beta$$

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Examples:

- $bird \mid\!\sim fly$. "Birds typically fly"
- $(tomato \mid\!\sim veg) \land (veg \mid\!\sim \neg fruit)$. "Tomatoes are usually considered vegetables, and vegetables are usually not fruits."

KLM Defeasible Reasoning: Semantics

$$\mathcal{K} = \{\mathit{penguin} \rightarrow \mathit{bird}, \mathit{bird} \mid\!\sim \mathit{fly}, \mathit{penguin} \mid\!\sim \neg \mathit{fly}\}$$

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Definition [1, 6]

A **ranking function** is a function $R : \mathcal{U} \rightarrow \mathbb{N} \cup \{\infty\}$, where $\mathcal{U}$ is the set of classical valuations over a set of atoms, such that if $R(u) < \infty$, then for every $0 \leq j < R(u)$ there exists $v \in \mathcal{U}$ such that $R(v) = j$.

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- Rank ∞ are "impossible" states (They break strict rules).
- Here, $bird \mathrel{|}\!\sim fly$ follows from the table since in the lowest rank where $bird$ is true, we have that $bird \rightarrow fly$.

Rational Closure

- In the context of KLM defeasible reasoning, there are several different notions of non-monotonic entailment from a knowledge base $\mathcal{K}$.
- We focus on **Rational Closure** [6], which is often considered one of the most fundamental notions of non-monotonic entailment in KLM-style reasoning.

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- Then, if r_{RC} entails $\alpha \mid\sim \beta$, we write $\mathcal{K} \mid\approx_{RC} \alpha \mid\sim \beta$.
- Given $\mathcal{K}$ and $\alpha \mid\sim \beta$, there is an equivalent algorithm which determines whether $\mathcal{K} \mid\approx_{RC} \alpha \mid\sim \beta$. The complexity of this is P^{NP}. [2, 6]

Standpoint Logics: Syntax

Propositional Standpoint Logic [4]

The language of **propositional standpoint logic** is a multi-modal logic where the modal operators are indexed by a set of standpoint symbols $\mathcal{S}$ representing agents:

$\Box_s\phi =$ "it is unequivocal to s that ϕ" $\Diamond_s\phi =$ "it is possible to s that ϕ"

We also include **standpoint sharpening statements** of the form $s_1 \preceq s_2$ which intuitively expresses that s_1 is a "more specific" standpoint to s_2.

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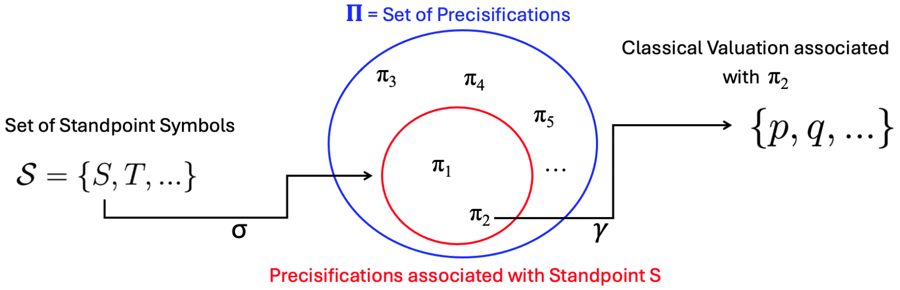
- $\Diamond_A(\mathit{fruit} \wedge \mathit{veg})$. "According to A, it is possible for something to be both a fruit and vegetable."

Standpoint Logics: Semantics

The semantics are given by a triple $M = (\Pi, \sigma, \gamma)$: [4]



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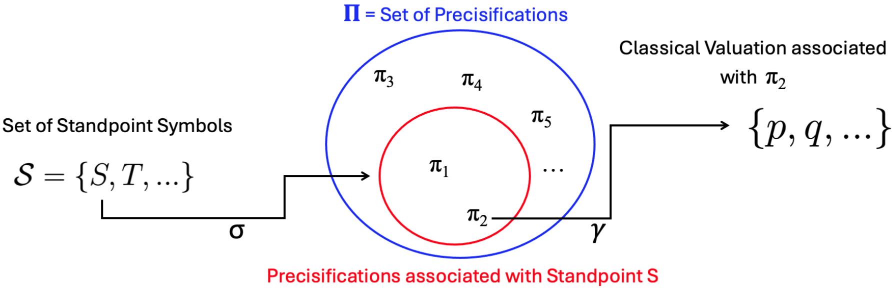

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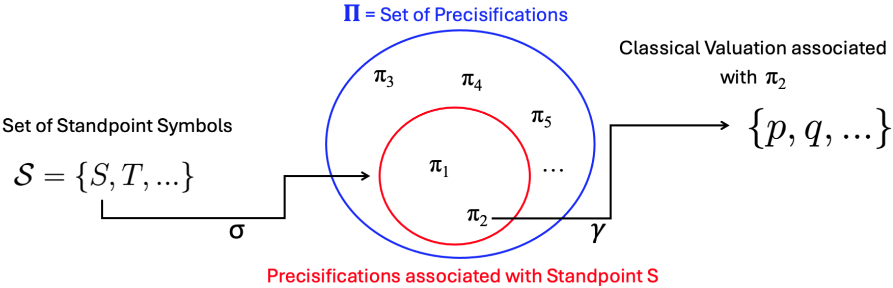

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Variation of a multi-modal S5.

Example

Two agents, A and B:

- $\Box_A(tomato \to fruit)$. "Tomatoes are fruits."
- $\Diamond_A(fruit \land veg)$. "It is possible for a fruit to be a vegetable."
- $\Box_B(tomato \to (fruit \lor veg))$. "Tomatoes are fruit or veg."
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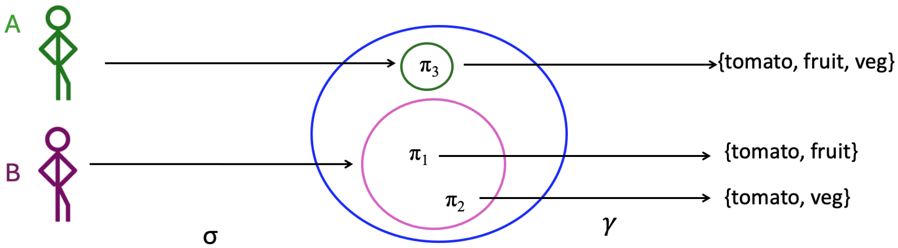
Model:



Defeasible Restricted Standpoint Logic

Language of DRSL

We allow statements of the form

$$\psi ::= \alpha \mid \alpha \mathrel{|\!\sim} \beta \mid \Box_s \psi \mid \Diamond_s \psi \mid \psi \wedge \psi \quad \text{or} \quad e ::= s \preceq t$$

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where α, β are Boolean formulas and s and t are standpoint symbols.

Extends propositional standpoint logic by allowing agents to hold *defeasible* beliefs.

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Going back to our motivating example:

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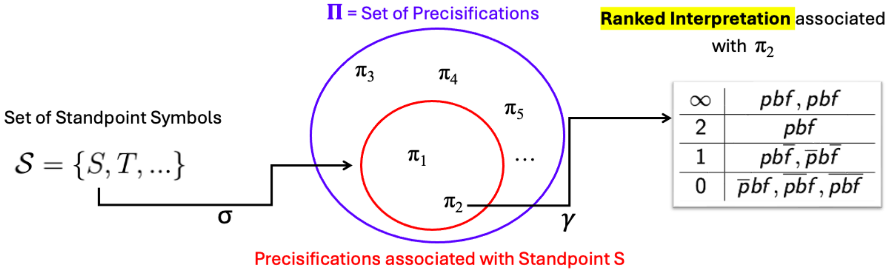
$$\{L \preceq C, \Box_L(fruit \rightarrow \neg veg)\}$$

Defeasible Restricted Standpoint Logic: Semantics

Looks very similar to the propositional case:



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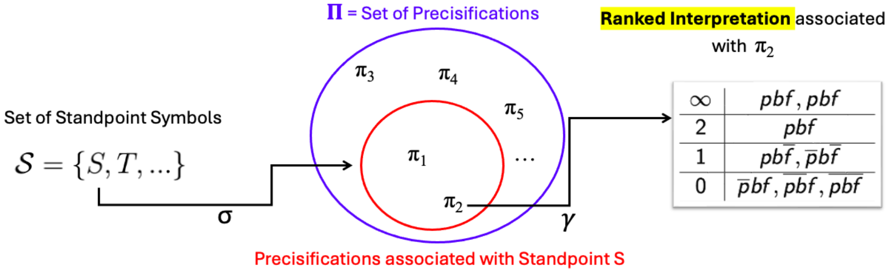
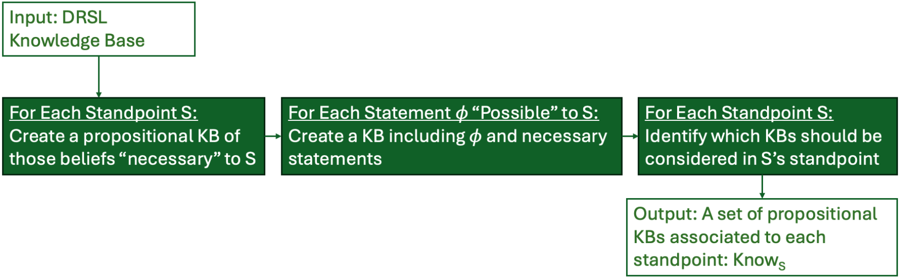


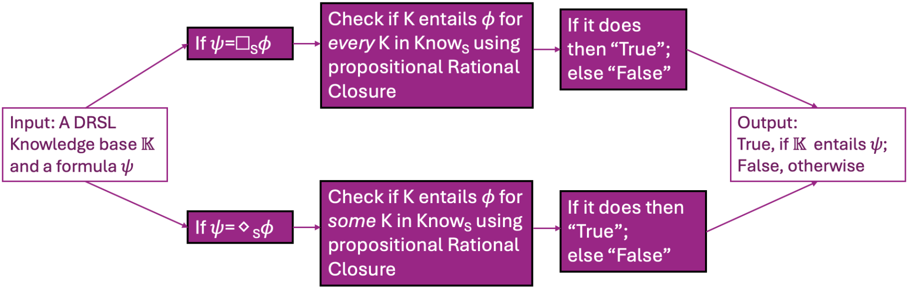
We just have a **ranking function** instead of a classical valuation.

Rational Closure Algorithm for DRSL

We split a DRSL knowledge base into several propositonal KLM ones:



We use this to check entailment:



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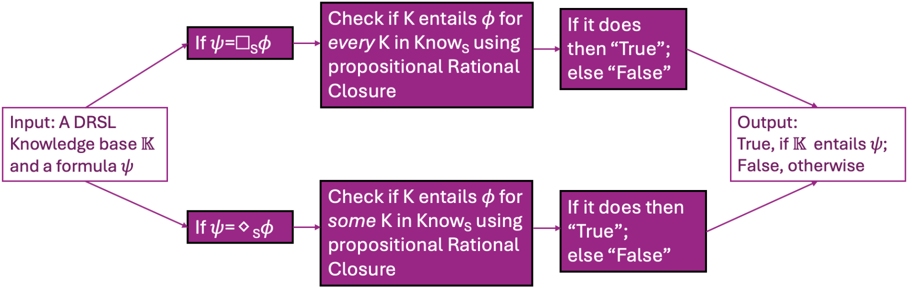
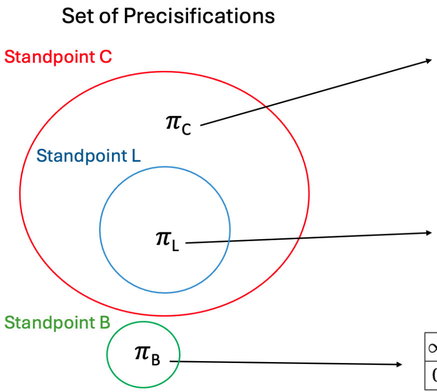


Theorem

The above algorithm is in P^{NP}, the same as entailment checking for propositional Rational Closure.

This Rational Closure can also be constructed using a single semantic structure. Using our motivating example:



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Semantics for Rational Closure

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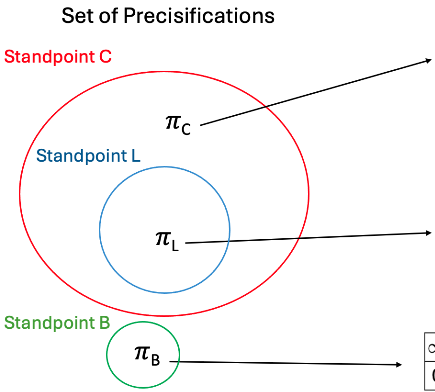

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We can use this to check conclusions like "$\Box_{All}(tomato \mid\!\sim veg)$"!

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- Proposed the language and semantics of DRSL to handle multiple standpoints who hold defeasible beliefs.
- Constructed a notion of (non-monotonic) entailment that extends Rational Closure from the propositional case.
- Main Result 1: Rational Closure has an equivalent definition using an algorithm or a single semantic model.
- Main Result 2: Entailment checking is computable in P^{NP}. Complexity is the same as in the propositional case!

Ongoing and Future Work

1. Develop a representation result with KLM-style postulates which capture the semantics.
2. Analyze different notions of entailment. (Eg. lexicographic closure, c-inference)
3. Belief revision in standpoint logics: How does it look when agents change their beliefs based on each other's beliefs?

Thank you!

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