Conditional Reasoning in KR

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Conditional Reasoning

- **Conditional Logics**
- Conditionals are statements of the form **If** A **then** B

$$\mathbf{A} \Rightarrow \mathbf{B}$$

- Developed to model our reasoning.
- A lot of attention in the last decades, especially connected with the development of AI.

Plan of the talk

- **Motivation**: why conditionals?
- **Basic formal ideas**
  - Conditional vs. entailment
- **Future directions**

Modern mathematical logic: 2 notions modeling IF-THEN relations:

1. **Material Implication** (inside the language):

$$\mathbf{A \rightarrow B}$$

- Logically equivalent to $\neg(A \wedge \neg B)$ and $\neg A \vee B$

A	B	$A \rightarrow B$
T	T	T
T	F	F
F	T	T
F	F	T

CONDITIONAL REASONING

Modern mathematical logic: 2 notions modeling IF-THEN relations:

2. **Classical entailment relation** (meta-linguistic):

$$\mathbf{A} \models \mathbf{B}$$

B is a **logical consequence** of *A* (*A* **entails** *B*)

if

in every situation in which *A* is true, also *B* is true.

Or, equivalently,

it cannot be the case that *A* is true and *B* is false.

Conditional Reasoning

Classical entailment is strictly connected to three structural properties:

- **Inclusion**: $\mathcal{K} \models A$ for every $A \in \mathcal{K}$;
- **Monotonicity**: If $\mathcal{K} \models A$, then $\mathcal{K} \cup \{B\} \models A$;
- **Cut**: If $\mathcal{K} \cup \{B\} \models A$ and $\mathcal{K} \models B$, then $\mathcal{K} \models A$.

- Strong connection between material implication and classical entailment implication:

$$A \models C \text{ iff } \models A \rightarrow C$$

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- **Inadequacy of the classical conditional outside math**

The proof of the existence of God proposed by Edgington [Edg95]:

If God does not exist, then it's not the case that, if I pray, my prayers will be answered	$\neg G \rightarrow \neg(P \rightarrow A)$
I do not pray	$\neg P$

- **Inadequacy of the classical conditional outside math**

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If God does not exist, then it's not the case that, if I pray, my prayers will be answered	$\neg G \rightarrow \neg(P \rightarrow A)$
I do not pray	$\neg P$
Therefore, God exists	G

$$\neg G \rightarrow \neg(P \rightarrow A), \neg P \models G$$

Conditional Reasoning

P	A	G	$\neg P$	$\neg(P \rightarrow A)$	$\neg G \rightarrow \neg(P \rightarrow A)$
T	T	T	F	F	T
T	T	F	F	F	F
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	F	T
F	T	F	T	F	F
F	F	T	T	F	T
F	F	F	T	F	F

Conditional Reasoning

P	A	G	$\neg P$	$\neg(P \rightarrow A)$	$\neg G \rightarrow \neg(P \rightarrow A)$
T	T	T	F	F	T
T	T	F	F	F	F
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	F	T
F	T	F	T	F	F
F	F	T	T	F	T
F	F	F	T	F	F

P	A	G	$\neg P$	$\neg(P \to A)$	$\neg G \to \neg(P \to A)$
T	T	T	F	F	T
T	T	F	F	F	F
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	F	T
F	T	F	T	F	F
F	F	T	T	F	T
F	F	F	T	F	F

Conditional Reasoning

Some less paradoxical examples [Pri18]:

- **Transitivity**: $A \rightarrow B, B \rightarrow C / A \rightarrow C$.
  - If Hoover had been born in Russia, he would have been a communist.
  - If Hoover had been a communist, he would have been a traitor.
  - Hence, if Hoover had been born in Russia, he would have been a traitor.
- **Monotonicity**: $A \rightarrow C / (A \wedge B) \rightarrow C$.
  - If you jump off a tall building, you will die.
  - Hence, if you jump off of a tall building and you are wearing a safety harness, you will die.

Everyday reasoning $\neq$ Mathematical reasoning

Commonsense conditionals $\neq$ Mathematical conditionals

Conditional Reasoning

- Propositional language $\mathcal{L} = \{A, B, \dots\}$
- Conditional language
  $\mathcal{L}^{\Rightarrow} = \{A \Rightarrow B, C \Rightarrow D \dots\} \cup \{A \rightarrow B, C \rightarrow D \dots\}$

- A Knowledge Base is a finite set of conditionals

$$\mathcal{K} = \{A_1 \Rightarrow B_1, A_2 \Rightarrow B_2 \dots\}$$

From a knowledge base we can derive new conditionals. E.g.,

$$\{bird \Rightarrow fly, eagle \rightarrow bird\} \mid\!\approx eagle \Rightarrow fly$$

$$\mathbf{A} \Rightarrow \mathbf{B}$$

Conditionals represent two aspects:

- The existence of a connection between a **condition** and an **effect** (both represented by propositions).
- The **modality** of such a connection. For example:
  - **Necessary**:
    "For every triple of natural numbers x, y, z, if $x > y$ and $y > z$, then $x > z$"
  - **Presumptive/Plausible**:
    "If it is a bird, then it presumably flies"

Conditional Reasoning

- **Probable**:
  "If you go out in this weather, you will probably get a cold"
- **Causal**:
  "If you throw a stone against that window, then you will break it"
- **Deontic**:
  "If you have had alcohol, you should not drive"
- **Counterfactual**:
  "If I were you, I wouldn't do that"
- ...

- **Reasoning** with conditionals: given a conditional KB $\mathcal{K} = \{A_1 \Rightarrow B_1, \ldots, A_n \Rightarrow B_n\}$, derive new conditionals $C \Rightarrow D$.

$$\mathcal{K} \mid\!\approx \mathbf{C} \Rightarrow \mathbf{D}$$

Two advantages of using conditionals for formalising reasoning is having a **general framework** s.t. we can:

- **model different kinds of reasoning patterns**, by modifying the semantics and the decision procedures;
- **compare** different kinds of reasoning.

The **basic reasoning patterns** can be modeled through **structural properties**.

(LLE) $$\frac{\models A \equiv B,\ A \Rightarrow C}{B \Rightarrow C}$$

(RLE) $$\frac{\models B \equiv C,\ A \Rightarrow B}{A \Rightarrow C}$$

(Ref) $$A \Rightarrow A$$

(Cut) $$\frac{\models A \wedge B \Rightarrow C,\ A \Rightarrow B}{A \Rightarrow C}$$

(And) $$\frac{A \Rightarrow B,\ A \Rightarrow C}{A \Rightarrow B \wedge C}$$

(Or) $$\frac{A \Rightarrow C,\ B \Rightarrow C}{A \vee B \Rightarrow C}$$

(RW) $$\frac{A \Rightarrow B,\ \models B \rightarrow C}{A \Rightarrow C}$$

(Mon) $$\frac{A \Rightarrow C,\ \models B \rightarrow A}{B \Rightarrow C}$$

(EFQ) $$\frac{\models \neg A}{A \Rightarrow B}$$

(Con) $$\frac{A \Rightarrow B,\ \models \neg B}{\models \neg A}$$

(Sup) $$\frac{\models A \rightarrow B}{A \Rightarrow B}$$

(CM) $$\frac{A \Rightarrow B,\ A \Rightarrow C}{A \wedge B \Rightarrow C}$$

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Conditional Reasoning - Structural Properties

- **Right Conjunction**.

$$\frac{A \Rightarrow B \quad A \Rightarrow C}{A \Rightarrow B \wedge C} \quad \text{(And)}$$

  - Horses are tall
  - Horses are fast
  - Hence, horses are tall and fast

- **Right Weakening**.

$$\frac{C \Rightarrow D \quad \models D \rightarrow E}{C \Rightarrow E} \quad \text{(RW)}$$

  - If you kill someone, you commit murder
  - Committing murder is committing a crime
  - If you kill someone, you commit a crime

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The **System P** [KLM90]

(LLE) $\dfrac{\models A \equiv B,\ A \Rightarrow C}{B \Rightarrow C}$

(RLE) $\dfrac{\models B \equiv C,\ A \Rightarrow B}{A \Rightarrow C}$

(Ref) $A \Rightarrow A$

(Cut) $\dfrac{\models A \wedge B \Rightarrow C,\ A \Rightarrow B}{A \Rightarrow C}$

(And) $\dfrac{A \Rightarrow B,\ A \Rightarrow C}{A \Rightarrow B \wedge C}$

(Or) $\dfrac{A \Rightarrow C,\ B \Rightarrow C}{A \vee B \Rightarrow C}$

(RW) $\dfrac{A \Rightarrow B,\ \models B \rightarrow C}{A \Rightarrow C}$

(EFQ) $\dfrac{\models \neg A}{A \Rightarrow B}$

(Sup) $\dfrac{\models A \rightarrow B}{A \Rightarrow B}$

(CM) $\dfrac{A \Rightarrow B,\ A \Rightarrow C}{A \wedge B \Rightarrow C}$

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Depending on the kind of reasoning, all the properties above can be debatable.

- **Deontic Reasoning**.

$A \Rightarrow B$ read as "If A, then it ought to be B"

Reflexivity (Ref),

$$A \Rightarrow A,$$

is not considered a desiderata anymore (by someone).

'If there is a crime, then there ought to be a crime'

Conditional Reasoning - Structural Properties

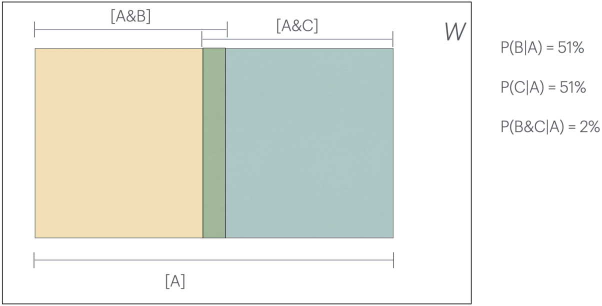
- **Threshold Probabilistic Reasoning**. For example,

$$A \Rightarrow B \text{ iff } \mathcal{P}(B \mid A) > 50\%$$

Right Conjunction (And),

$$\frac{A \Rightarrow B, \quad A \Rightarrow C}{A \Rightarrow B \wedge C},$$

is not a desiderata anymmore.



CONDITIONAL REASONING - SEMANTICS

Which **truth conditions** for evaluating **A** $\Rightarrow$ **B**?

Consider situations in which *A* is true

$\Downarrow$

check if *B* is true too.

- **Mathematical reasoning**: Refer to **all** the situations in which *A* is true.
- **Everyday reasoning**: Refer to **some** of the situations in which *A* is true.

Which ones?

- **Main idea**:
  - Define some kind of priority among the possible situations;
  - given $A \Rightarrow B$, its **confirmation** $A \wedge B$ is **preferred to** its **refutation** $A \wedge \neg B$.

An intuitive and popular possible-world semantics: **Ranked Interpretations**

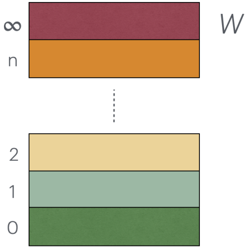


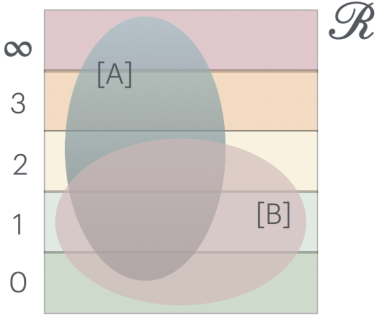
FIGURE: Ranked interpretation

where $w \prec v$ iff w is preferred to v.

- **Satistaction**:

$$\mathcal{R} \Vdash A \Rightarrow B \text{ iff } \min_{\prec}([A]) \subseteq [B]$$

where $[A] =$
$\{w \text{ in } \mathcal{R} \text{ s.t. } A \text{ is satisfied in } w\}$



Conditional Reasoning - Semantics

Other (possible-worlds) semantics:

- Preferential interpretations [Sho88, KLM90];
- Ranking functions [Spo12];
- Spheres [Gro88];
- Choice functions [Rot01];
- Possibilistic interpretations [DP14];
- Probabilistic interpretation [HP03];
- ...

Some of these semantics are very close to each other.

Monotonicity can be on two levels.

In mathematical logic we have e monotonic implication:

If $\mathbf{A} \rightarrow \mathbf{B}$ holds, then $(\mathbf{A} \wedge \mathbf{C}) \rightarrow \mathbf{B}$ holds.

and a monotonic entailment relation:

If $\{\mathbf{A_1}, \ldots, \mathbf{A_n}\} \models \mathbf{B}$ holds, then $\{\mathbf{A_1}, \ldots, \mathbf{A_n}, \mathbf{C}\} \models \mathbf{B}$ holds.

In our conditional language, we can have monotonicity at the level of conditionals:

$$\text{If } \mathbf{A} \Rightarrow \mathbf{B} \text{ holds, then } (\mathbf{A} \wedge \mathbf{C}) \Rightarrow \mathbf{B} \text{ holds.}$$

and a monotonicity at the level of the entailment relation:

If $\{\mathbf{A_1} \Rightarrow \mathbf{B_1}, \ldots, \mathbf{A_n} \Rightarrow \mathbf{B_n}\} \mid\!\approx \mathbf{C} \Rightarrow \mathbf{D}$ holds,
then $\{\mathbf{A_1} \Rightarrow \mathbf{B_1}, \ldots, \mathbf{A_n} \Rightarrow \mathbf{B_n}, \mathbf{A_{n+1}} \Rightarrow \mathbf{B_{n+1}}\} \mid\!\approx \mathbf{C} \Rightarrow \mathbf{D}$ holds.

Conditional Reasoning - Entailment relations

In conditional reasoning we have non-monotonic conditionals:

It can be **A** $\Rightarrow$ **B** holds, but **A** $\wedge$ **C** $\Rightarrow$ **B** does not.

What about entailment?

- We can have a entailment $\models\!\approx$ defined in the classical way:

$$\mathcal{K} \models\!\approx A \Rightarrow B \text{ iff } \mathcal{R} \Vdash A \Rightarrow B \text{ for } \textbf{every} \text{ model } \mathcal{R} \text{ of } \mathcal{K}.$$

Applied to preferential or ranked interpretations = **System P.**

Such an entailment relation is monotone:

$$\mathcal{K} \models\!\approx A \Rightarrow B \text{ implies } \mathcal{K}' \models\!\approx A \Rightarrow B \text{ for every } \mathcal{K}' \text{ s.t. } \mathcal{K}' \supseteq \mathcal{K}.$$

Conditional Reasoning - Entailment relations

Are monotonic entailment relations interesting for conditional reasoning?

- (Generally) **No!**

A monotonic entailment relation does not allow modeling interesting reasoning patterns. For example,

- **Presumption of non-exceptionality**:[1]

  If we are not aware that we are in an exceptional situation, we reason monotonically.

[1] Called *Presumption of Typicality* in [Leh95].

For example, assume you have a KB $\mathcal{K}$ containing only:

- Typically, birds fly ($\mathbf{B} \Rightarrow \mathbf{F}$);
- Squaphts are birds ($\mathbf{S} \rightarrow \mathbf{B}$).

Not having any other information about squaphts, we want to treat them as normal birds and we conclude that squaphts, presumably, fly

- $\mathcal{K} \mathrel{|\!\approx} \mathbf{S} \Rightarrow \mathbf{F}$.

Conditional Reasoning - Entailment relations

Assume that later we are informed that squaphts are some strange birds that do not fly. Hence now our KB is

$$\mathcal{K}' = \{\mathbf{B} \Rightarrow \mathbf{F}, \mathbf{S} \rightarrow \mathbf{B}, \mathbf{S} \Rightarrow \neg\mathbf{F}\}$$

But, being $\approx$ monotone, we have

- $\mathcal{K}' \models S \Rightarrow \neg F$ by Reflexivity, and
- $\mathcal{K}' \models S \Rightarrow F$ by Monotonicity.

Squaphts would be flying and not flying at the same time...

Most of the interesting conditional reasoning systems are **defeasible** on both levels, in the sense of being **non-monotonic** both at the level of **conditionals** and the level of **entailment**.

Non-monotonicity at the level of entailment implies that, given a KB $\mathcal{K}$, $\mathrel{|}\!\approx$ is defined by referring to the conditionals satisfied

- **NOT** by all the models of $\mathcal{K}$,
- but by some (usually **one**) model of $\mathcal{K}$.

Given a KB $\mathcal{K}$, we pick one specific model $\mathcal{R}$ of $\mathcal{K}$:

$$\mathcal{K} \models A \Rightarrow B \text{ iff } \mathcal{R} \Vdash A \Rightarrow B$$

The choice of $\mathcal{R}$ is determined by the kind of reasoning we want to model.

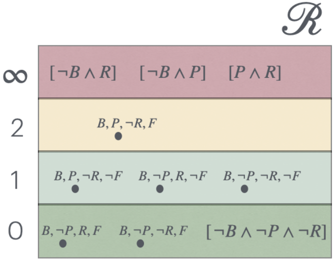
Conditional Reasoning - Entailment relations

- **Example:** the basic entailment relation $\approx$ created to model the **presumption of non-exceptionality** is the **Rational Closure** [LM92, Pea90, GGOP15]:

Given a KB $\mathcal{K}$, we consider the model of $\mathcal{K}$ in which all the worlds are "pushed" as low as possible.

Consider a "Penguin KB" (P is *penguin*, R is *robin*):

$\mathcal{K} = \{B \Rightarrow F, P \rightarrow B, R \rightarrow B,$
$P \rightarrow \neg R, P \Rightarrow \neg F\}$



$\mathcal{R} \Vdash R \Rightarrow F \qquad \mathcal{R} \Vdash P \Rightarrow \neg F \qquad \mathcal{R} \nVdash P \Rightarrow F$

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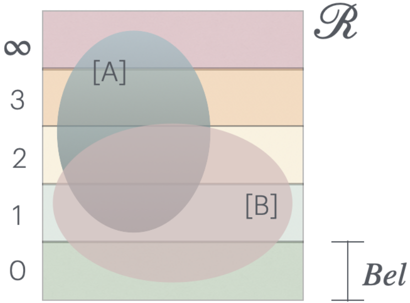
CONDITIONAL REASONING - ENTAILMENT RELATIONS

- Ideal workflow to define a new reasoning system for conditional reasoning:
  - Identify the **properties** of the kind of **reasoning** we want to model;
  - Choose a promising **semantic framework**;
  - Define what kind of **model** to use to determine the **entailment relation**;
  - Define **decision procedures** that are **correct** and **complete** w.r.t. the entailment relation;
  - Check the entailment relation is **adequate** w.r.t. the desired reasoning and it is **implementable**.

Conditional Reasoning and BR

Very strong connection with **Belief Revision**.



$$R \Vdash A \Rightarrow B \text{ iff } B \in (Bel * A)$$

Conditional Reasoning - Entailment relations

Some systems for conditional reasoning:

- Rational Closure/System Z [LM92, Pea90, GGOP15];
- Lexicographic Closure [Leh95];
- Inheritance-Based Closure [CS13a];
- Relevance Closure [CMMN14];
- Multi-preferential Closure [GG21];
- c-inferences [Ker01];
- System W [KB22].

FUTURE DIRECTIONS

Some possible future research directions

- Novel conditional **reasoning systems**.
- Consider **new languages**.
- **Implementations**.
- Interchange with **Cognitive Science**.
- Role in **explainable AI**.

These lines of research often intertwine.

Defining new forms of inferences, appropriate for modeling specific reasoning patterns.

Example: The problem of **relevance**.

- The **antecedent** of a conditional must provide a **reason** to accept its **consequent**.
- The problem of relevance goes beyond non-monotonicity.

  For example, problems with (RW),

$$\frac{C \Rightarrow D \quad \models D \rightarrow E}{C \Rightarrow E} \quad \text{(RW)}$$

Novel conditional reasoning systems

Some kinds of conditional reasoning that are uncomfortable with (RW):

- Deontic Reasoning:

• If you are involved in a car accident, you should remain on the spot ✓
• If you are involved in a car accident, you should remain on the spot or paint yourself in blue **x**

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NOVEL CONDITIONAL REASONING SYSTEMS

Some kinds of conditional reasoning that are uncomfortable with (RW):

- Deontic Reasoning:

• If you are involved in a car accident, you should remain on the spot ✓
• If you are involved in a car accident, you should remain on the spot or paint yourself in blue **x**

- Causal Reasoning:

• If you throw a stone against the window, it will break ✓
• If you throw a stone against the window, it will break or Ann will drink tea **x**

Novel conditional reasoning systems

- Rott's **Difference-Making Conditionals** [Rot22]:
  Given a 'standard' conditional $\Rightarrow$ we can define a new conditional $\Rrightarrow$ as

$$\mathbf{A} \Rrightarrow \mathbf{B} \text{ is accepted}$$
$$\text{iff}$$
$$\mathbf{A} \Rightarrow \mathbf{B} \text{ is accepted and } \neg\mathbf{A} \Rightarrow \mathbf{B} \text{ is } \textbf{not}.$$

$\mathbf{A} \Rrightarrow \mathbf{B}$ can be read as 'If A then **relevantly** B'.

Note: this is **not** the definition given by Rott, that refers to belief revision and the Ramsey test.

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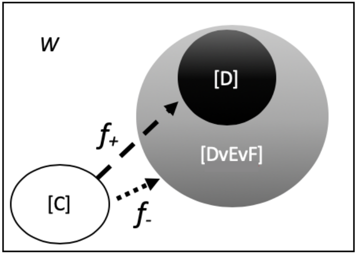
- **Basic Conditionals** [CMV19].

Effect Function: $\mathbf{f}_+ : 2^{\mathcal{W}} \longrightarrow 2^{(2^{\mathcal{W}})}$.

Relevance Function: $\mathbf{f}_- : 2^{\mathcal{W}} \longrightarrow 2^{(2^{\mathcal{W}})}$.

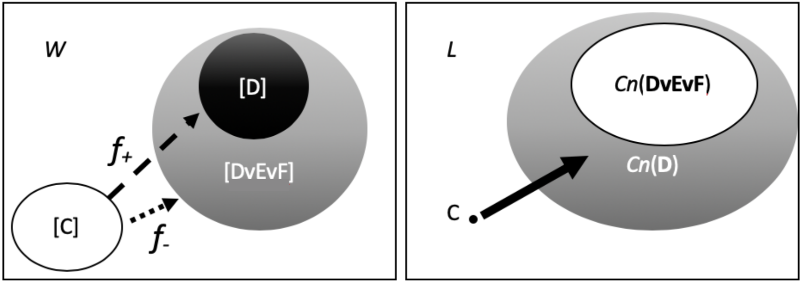
Relevance Interpretation: $\mathcal{I} = (\mathcal{W}, \mathbf{f}_+, \mathbf{f}_-)$.



$\pm$-satisfaction:

$\mathcal{I}$ $\pm$-satisfies $C \Rightarrow D$ ($\mathcal{I} \Vdash_{\pm} C \Rightarrow D$) if

- there is a $V \in f_{+}(C)$ s.t. $V \subseteq [D]_{\mathcal{W}}$; and
- for every $V' \in f_{-}(C)$, $V' \nsubseteq [D]_{\mathcal{W}}$.



Some approaches drop the possible-world semantics. For example:

- **Input/Output Logics** [MvdT00].
  Rules $(\mathbf{A}, \mathbf{B})$ with an **operational semantics**.

- **Conditional interpretations** $\mathcal{I} = (f, g)$ [CS22].

$$\begin{tikzcd}
A \arrow[r, "f"] & B' \arrow[d, "\leq"] \\
& B \arrow[ul, "g"]
\end{tikzcd}$$

Other languages

Conditional reasoning mainly developed for propositional logics.

Necessary to extend the reasoning systems to other languages, of particular interest for KR. For example:

- **Description Logics** [GGOP15, CMV23, PT18]
- **RDFS** [CS23]
- More expressive fragments of **FOL** [KT12, CMPV22]

- **Desiderata**:
  - **Adapt** the reasoning systems to the **new expressivity**.
  - **Computational costs** of the decision problems are in line with the computational costs of the classical reasoning.
  - Plus: **ease of implementation**. For example, the defeasible reasoner can be built on top of a classical reasoner.

Implement!

(please)

Interchange with Cognitive Science

- Humans present reasoning patterns that are successful, but cannot be modeled and justified using classical logic and classical probability theory.
- It is essential to choose the right logical framework in order to understand and model human reasoning adequately.
- **Cognitive Logics**[2]: formal, logic-based approaches to reasoning that are able to model human reasoning behaviour even if this is in conflict with (classical) logical standards.

[2]https://cognitive-logics.org/

Interchange with Cognitive Science

- It seems possible to explain and formalise reasoning behaviours that are not in line with classical logic and probability theory by **referring to conditional logics** [EKIR18].
- Conditional reasoning allows to consider two elements that are fundamental in commonsense reasoning:
  - **Context**
  - **Relevance**

Explainable AI

- **Conditionals** can be used as tools for explanations. In particular, **counterfactuals**.

- Counterfactual conditionals: “If A had not occurred, C would not have occurred” can explain causal claims.
  - Loan application: If Debt was none, then decision would not have been Reject
  - Risk of Heart Failure: If Smoking was No, then risk would not have been Increased

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Explainable AI

- Finding counterfactual explanations is a challenging search problem [LP24].
- Possible roles of reasoning:
  - derive new conditionals from the ones already found: promising candidates to be checked.
  - restrict the space of search.
- Useful tool to be developed:
  - **Multi-valued defeasible conditionals** [CS13b, ABB^{+}24]

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Thank you!

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