

# Bridging DLs and SHACL Validation

A DL perspective

---

Magdalena Ortiz

TU Wien

Joint work with Anouk Oudshoorn, Mantas Šimkus, Shqiponja Ahmetaj

# Table of contents

1. Introduction to Description Logics

2. DLs, OWA and the Real World

3. Introduction to SHACL

    Recursion in SHACL

4. Validating SHACL with Ontologies

    Semantics of SHACL w.r.t. Ontologies

    Validation via Rewriting

# Introduction to Description Logics

---

# Description Logics (DLs)

- Family of logics well-suited for describing structured knowledge through *concepts* (classes) and *roles* (relationships)
- **Decidable fragments of first order logic** (with a funny\* syntax)

# Description Logics (DLs)

- Family of logics well-suited for describing structured knowledge through **concepts** (classes) and **roles** (relationships)
- **Decidable fragments of first order logic** (with a funny\* syntax)

AfricanCountry	$\sqsubseteq$	Country
Country	$\sqsubseteq$	$\exists \text{ hasCapital.City}$
City	$\sqsubseteq$	$\exists \text{ locatedIn.Country}$
City	$\sqsubseteq$	$\leq 1 \text{ locatedIn.Country}$
AfricanCountry	$\equiv$	$\text{Country} \sqcap \exists \text{ locatedIn.}\{\text{Africa}\}$
hasCapital <sup>-</sup>	$\sqsubseteq$	locatedIn

*AfricanCountry(SouthAfrica), Country(Botswana), locatedIn(Botswana, Africa)*

# Description Logics (DLs)

- Family of logics well-suited for describing structured knowledge through **concepts** (classes) and **roles** (relationships)
- **Decidable fragments of first order logic** (with a funny\* syntax)

AfricanCountry	$\sqsubseteq$	Country
Country	$\sqsubseteq$	$\exists \text{ hasCapital.City}$
City	$\sqsubseteq$	$\exists \text{ locatedIn.Country}$
City	$\sqsubseteq$	$\leq 1 \text{ locatedIn.Country}$
AfricanCountry	$\equiv$	$\text{Country} \sqcap \exists \text{ locatedIn}\{\text{Africa}\}$
hasCapital <sup>-</sup>	$\sqsubseteq$	locatedIn

*AfricanCountry(SouthAfrica), Country(Botswana), locatedIn(Botswana, Africa)*

- DL **knowledge base** = **TBox** (axioms) + **ABox** (data)

\* Yes, it is an acquired taste.

# DLs as FOL Fragments

- guarded quantification, no explicit variables, function-free fragments
- mostly contained in  $\text{FO}^2$  or  $\mathcal{C}^2$

KBs are theories with two types of formulas:

- **ABox**: data, facts  $B(c), B(d), r(c, d), r_2(d, e) \dots$
- **TBox**: axioms, universal formulas

$A \sqcap B \sqsubseteq C$	$\forall x A(x) \wedge B(x) \rightarrow C(x)$
$A \sqsubseteq \exists r.B$	$\forall x A(x) \rightarrow \exists y r(x, y) \wedge B(y)$
$C \sqsubseteq \forall r.\{a\}$	$\forall x, y C(x) \wedge r(x, y) \rightarrow y = a$
$r \sqsubseteq s$	$\forall x, y r(x, y) \rightarrow s(x, y)$

# DLs: a toolbox of Computational Logics

DLs are a **toolbox**:

- different **constructors** combine into **many DLs**
- **decidable fragments** of standard FOL
- choice of the **right logic**
- taking into account the **computational cost**

			DL-Lite	$\mathcal{EL}$	$\mathcal{ALC}$	$\mathcal{SHOIQ}$
AfricanCountry	$\sqsubseteq$	Country	✓	✓	✓	✓
Country	$\sqsubseteq$	$\forall \text{hasCapital.City}$			✓	✓
Country	$\sqsubseteq$	$\neg \text{City}$	✓	✓	✓	✓
City $\sqcap \exists \text{locIn.AfrCntr}$	$\sqsubseteq$	AfricanCity		✓	✓	✓
SouthAfrica	$\sqsubseteq$	$\exists \text{locatedIn}\{Africa\}$				
hasCapital <sup>-</sup>	$\sqsubseteq$	locatedIn	✓			✓
		$\text{trans}(\text{locatedIn})$				✓



## DLs, OWA and the Real World

---

# What are DLs good for?

What have we achieved in four decades?

- A rich family of logics to describe and reason about graph shaped data
- Good understanding of expressiveness vs. complexity trade-off
- Boundaries of decidability
- Fragments, safe combinations
- A whole arsenal of algorithmic techniques
- Efficient reasoners
- ...

# The classic DL Setting

- Data as a labeled graph (ABox)
- Background knowledge in an ontology or TBox

# The classic DL Setting

- Data as a labeled graph (ABox)
- Background knowledge in an ontology or TBox

**Models** as in classical predicate logic

- any structure that satisfies the theory
- for us: any extension of the ABox that satisfies the TBox

*VeggiePizza(margherita)      hasTopping(margherita, mozzarella)*

*VeggiePizza  $\sqsubseteq$  Pizza*

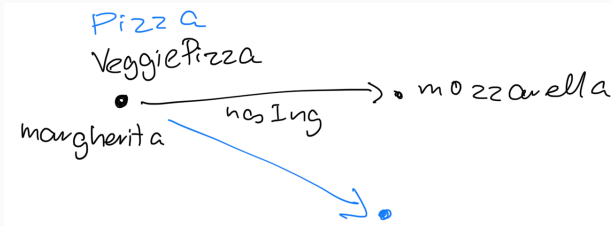
*Pizza  $\sqsubseteq_{\geq 2}$  hasTopping.T*

# DL KB Examples

$\text{VeggiePizza}(\text{margherita}) \quad \text{hasTopping}(\text{margherita}, \text{mozzarella})$

$\text{VeggiePizza} \sqsubseteq \text{Pizza}$

$\text{Pizza} \sqsubseteq_{\geq 2} \text{hasTopping.T}$



**Models** : any extension of the ABox that satisfies the TBox

**Models** : any extension of the ABox that satisfies the TBox

Typically, reason over **all models**, or similarly, **model existence**

- **Logical entailment**: subsumption, instance checking, query answering
- **consistency**, ...

In practice, (too) **many models!**

- costly reasoning
- some are unintended!
- sometimes misused or misunderstood



# DL Reasoning Examples

*Pizza(margherita)*

*hasTopping(margherita, mozzarella)*

*hasTopping(margherita, tomato)*

*VeggieTopping(mozzarella)*

*VeggieTopping(tomato)*

*Pizza*  $\sqcap \forall \text{hasTopping.VeggieTopping} \sqsubseteq \text{VeggiePizza}$

## OWL is too hard #56



dbooth-boston opened this issue on Mar 12, 2019 · 15 comments

### **You probably don't need OWL**

And if you do there's a simple way to prove it.

Ultimately you may be right, that OWL will never be easy enough for the middle 33%, *because* it is rooted in the OWA, and middle-33%-ers should just avoid it. But I still think it is worthwhile to challenge the community to see if we can do better, to reach a broader user base.



r/semanticweb · 1 yr. ago  
mfairview

### **Why I Don't Use OWL Anymore**

# Wrong expectations

```
:Library rdfs:subClassOf [  
  a owl:Restriction ;  
  owl:allValuesFrom :Book ;  
  owl:onProperty :holds  
] ;
```



This is a ridiculously convoluted way to say that a :Library instance has a :holds property with zero or more :Book values.

Let me start by saying that `rdfs:domain` and `rdfs:range` are not constraints and don't mean what you imply. They are merely producing inferences. Having said this, many people have in the past used them to "mean" constraints simply because there was no other modeling language. For background, see

Attempts to use DLs as **constraint languages** to describe data!

**SHACL**     <https://www.w3.org/TR/shacl/>

- SHApE Constraint Language
- W3C standard since 2017
- For RDF data

**SHACL**     <https://www.w3.org/TR/shacl/>

- SHaPE Constraint Language
- W3C standard since 2017
- For RDF data

**DL syntax with constraint semantics**

# Introduction to SHACL

---

# Basic SHACL Syntax

Vocabulary:

- set of **property names**  $P$  (aka roles)
- set of **class names**  $C$  (aka concepts)
- set of **node names**  $N$  (aka individuals)

We define **shape expressions**  $\varphi$ :

$$\varphi ::= A \mid \top \mid \{a\} \mid \neg\varphi \mid \varphi \wedge \varphi \mid \geq_n E.\varphi \mid E = E$$

where  $A \in C$ ,  $a \in I$ ,  $n \geq 0$ , and  $E$  is a **path expression**

$$E ::= p \mid p^- \mid E \cup E \mid E \circ E \mid E^*$$

with  $p \in P$ .

As usual, we can express

$$\varphi_1 \vee \varphi_2 \quad \exists E.\varphi \quad \forall E.\varphi \quad \leq_{n-1} E.\varphi$$

## Example Shape Expressions

$\text{Pizza} \wedge \exists \text{hasTopping}.\{\text{mozzarella}\} \wedge \forall \text{hasTopping}.\text{Vegetarian}$

$\text{Pizza} \wedge \geq_2 \text{hasTopping}.\{\text{aubergine}\} \vee \{\text{onion}\} \vee \{\text{artichoke}\}$



# SHACL Validation - basics

Input:

1. A **data graph**  $G$
2. A **shape expression**  $\varphi$
3. A set of **target nodes**  $a_1, \dots, a_n$

# SHACL Validation - basics

Input:

1. A **data graph**  $G$
2. A **shape expression**  $\varphi$
3. A set of **target nodes**  $a_1, \dots, a_n$

The **target nodes** may be e.g., nodes in a class or complex query

# SHACL Validation - basics

Input:

1. A **data graph**  $G$
2. A **shape expression**  $\varphi$
3. A set of **target nodes**  $a_1, \dots, a_n$

The **target nodes** may be e.g., nodes in a class or complex query

Question: **Does  $G$  validate  $\varphi$  at each  $a_i$ ?**

- View the **data graph as an interpretation**
  - Semantics of connectives in shape expressions as usual
- **Is  $a_i$  in the extension of  $\varphi$ ?, i.e., is  $G$  a model of  $a_i : \varphi$ ?**

# SHACL Validation - basics

Input:

1. A **data graph**  $G$
2. A **shape expression**  $\varphi$
3. A set of **target nodes**  $a_1, \dots, a_n$

The **target nodes** may be e.g., nodes in a class or complex query

Question: **Does  $G$  validate  $\varphi$  at each  $a_i$ ?**

- View the **data graph as an interpretation**
  - Semantics of connectives in shape expressions as usual
- **Is  $a_i$  in the extension of  $\varphi$ ?, i.e., is  $G$  a model of  $a_i : \varphi$ ?**

Feasible in PTime

- **Model checking** is typically easy!

# SHACL Validation - basics

Input:

1. A **data graph**  $G$
2. A **shape expression**  $\varphi$
3. A set of **target nodes**  $a_1, \dots, a_n$

The **target nodes** may be e.g., nodes in a class or complex query

Question: **Does  $G$  validate  $\varphi$  at each  $a_i$ ?**

- View the **data graph as an interpretation**
  - Semantics of connectives in shape expressions as usual
- **Is  $a_i$  in the extension of  $\varphi$ ?, i.e., is  $G$  a model of  $a_i : \varphi$ ?**

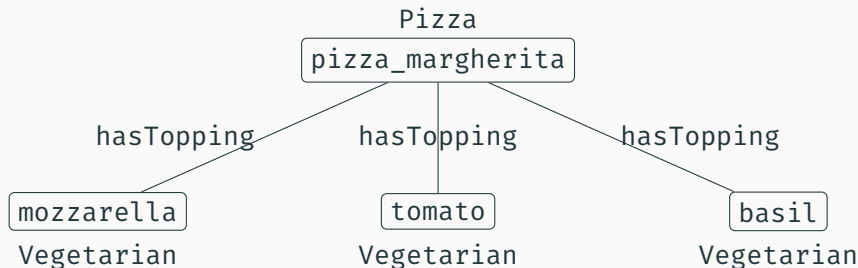
Feasible in PTime

- **Model checking** is typically easy!

**Satisfiability and containment** of SHACL constraints are **undecidable!**

# Validation on an Example

Given the graph:



This shape expression validates for the target node `margherita`:

$$\text{Pizza} \wedge \forall \text{hasTopping.Vegetarian}$$

# Shape names in SHACL

The SHACL vocabulary has more:

- set of **property names P** (aka roles)
- set of **node names N** (aka individuals)
- set of **class names C** (aka concepts)
- set of **shape names S** (similar to concepts)

# Shape names in SHACL

The SHACL vocabulary has more:

- set of **property names P** (aka roles)
- set of **node names N** (aka individuals)
- set of **class names C** (aka concepts)
- set of **shape names S** (similar to concepts)

In SHACL **constraints**, we assign **shape expressions** to **shape names**

$$S \leftarrow \varphi$$



# Shape names in SHACL

The SHACL vocabulary has more:

- set of **property names**  $P$  (aka roles)
- set of **node names**  $N$  (aka individuals)
- set of **class names**  $C$  (aka concepts)
- set of **shape names**  $S$  (similar to concepts)

In SHACL **constraints**, we assign **shape expressions** to **shape names**

$$S \leftarrow \varphi$$

**Targets** can then be given as sets of **shape atoms**

$$s_1(a_1), \dots, s_n(a_n)$$

A **shapes graph** contains **constraints**  $\mathcal{C}$  and **targets**  $\mathcal{T}$

The SHACL specification allows **shape names** in shape expressions:

$$\varphi ::= s \mid A \mid \top \mid \{a\} \mid \neg\varphi \mid \varphi \wedge \varphi \mid \geq_n E.\varphi \mid E = E$$

# Recursion in SHACL

The SHACL specification allows **shape names** in shape expressions:

$$\varphi ::= s \mid A \mid T \mid \{a\} \mid \neg\varphi \mid \varphi \wedge \varphi \mid \geq_n E.\varphi \mid E = E$$

*Pizza*  $\leftarrow \geq_2 \text{hasTopping}.T,$

*VeggiePizza*  $\leftarrow \text{Pizza} \wedge \forall \text{hasTopping}.\text{VeggieTopping},$

*VeggieTopping*  $\leftarrow \{\text{mozzarella}\} \vee \{\text{tomato}\} \vee \{\text{basil}\} \vee \{\text{artichoke}\}$

*Person*  $\leftarrow \exists \text{hasParent}.\text{Person}$

We assume each  $s$  occurs in only one constraint head:

$$s \leftarrow \varphi_1 \vee \cdots \vee \varphi_n$$

We now need a **shape assignment**  $\mathcal{I}$  that assigns a set of nodes  $s^{\mathcal{I}}$  to each shape name  $\mathcal{I}$ .

We assume each  $s$  occurs in only one constraint head:

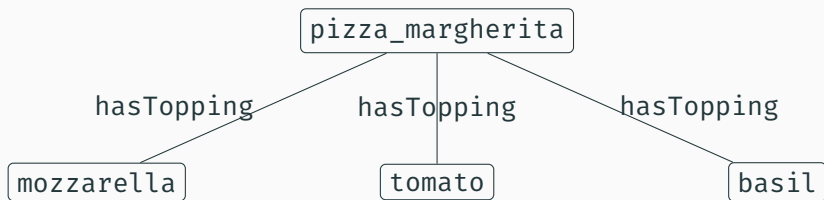
$$s \leftarrow \varphi_1 \vee \cdots \vee \varphi_n$$

We now need a **shape assignment**  $\mathcal{I}$  that assigns a set of nodes  $s^{\mathcal{I}}$  to each shape name  $\mathcal{I}$ .

**Validation** of  $(\mathcal{C}, \mathcal{T})$  consists on finding an assignment  $\mathcal{I}$  such that:

- $s^{\mathcal{I}} = \varphi^{\mathcal{I}}$  for each constraint  $s \leftarrow \varphi$  in  $\mathcal{C}$
- $a \in s^{\mathcal{I}}$  for every target  $s(a)$  in  $\mathcal{T}$

## Back to our Example



$Pizza \leftarrow \geq_2 \text{hasTopping}.T,$

$VeggiePizza \leftarrow Pizza \wedge \forall \text{hasTopping}.VeggieTopping,$

$VeggieTopping \leftarrow \{\text{mozzarella}\} \vee \{\text{tomato}\} \vee \{\text{basil}\} \vee \{\text{artichoke}\}$

Validation of  $(\mathcal{C}, \mathcal{T})$  consists on **finding an assignment  $\mathcal{I}$**  such that:

- $s^{\mathcal{I}} = \varphi^{\mathcal{I}}$  for each constraint  $s \leftarrow \varphi$  in  $\mathcal{C}$
- $a \in s^{\mathcal{I}}$  for every target  $s(a)$  in  $\mathcal{T}$

# Semantics of recursive SHACL

Validation of  $(\mathcal{C}, \mathcal{T})$  consists on **finding an assignment**  $\mathcal{I}$  such that:

- $s^{\mathcal{I}} = \varphi^{\mathcal{I}}$  for each constraint  $s \leftarrow \varphi$  in  $\mathcal{C}$
- $a \in s^{\mathcal{I}}$  for every target  $s(a)$  in  $\mathcal{T}$

**Satisfiability in DLs** where:

- property names **P** and node names **N** are **closed**
- shape names **S** are **open**



# Semantics of recursive SHACL

Validation of  $(\mathcal{C}, \mathcal{T})$  consists on **finding an assignment**  $\mathcal{I}$  such that:

- $s^{\mathcal{I}} = \varphi^{\mathcal{I}}$  for each constraint  $s \leftarrow \varphi$  in  $\mathcal{C}$
- $a \in s^{\mathcal{I}}$  for every target  $s(a)$  in  $\mathcal{T}$

**Satisfiability in DLs** where:

- property names **P** and node names **N** are **closed**
- shape names **S** are **open**

Constraints as concept definitions  $s \equiv \varphi$ , targets as concept assertions  $s(a)$

## Finding an assignment $\mathcal{I}$

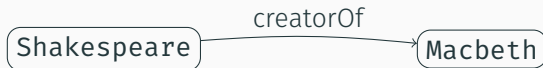
- makes satisfiability coNP hard
- are all assignments equally good?

*Director*  $\leftarrow \exists \text{creatorOf} . \text{Movie}$

*Movie*  $\leftarrow \exists \text{creatorOf}^- . \text{Director}$

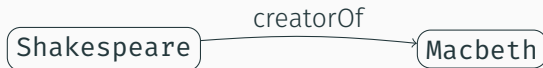
*Director*  $\leftarrow \exists \text{creatorOf} . \text{Movie}$

*Movie*  $\leftarrow \exists \text{creatorOf}^- . \text{Director}$



$Director \leftarrow \exists creatorOf.Movie$

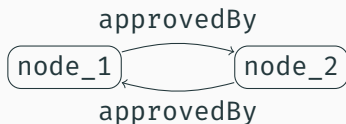
$Movie \leftarrow \exists creatorOf^-.Director$



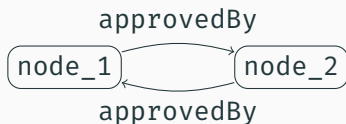
We can validate  $Director(Shakespeare)$  !?!

*certifiedNode*  $\leftarrow \exists \text{granted.Certifate} \vee \exists \text{approvedBy.certifiedNode}$

*certifiedNode*  $\leftarrow \exists \text{granted.Certificate} \vee \exists \text{approvedBy.certifiedNode}$



$certifiedNode \leftarrow \exists granted.Certificate \vee \exists approvedBy.certifiedNode$



We can validate *certifiedNode*(`node_1`)

# Refining the Semantics of Recursive SHACL

- **Supported semantics**: any shape assignment
- Other semantics choose **well-founded assignments**



# Refining the Semantics of Recursive SHACL

- **Supported semantics**: any shape assignment
- Other semantics choose **well-founded assignments**

## DLs in 1990s

descriptive vs. least fixpoint semantics for terminological cycles

# Refining the Semantics of Recursive SHACL

- **Supported semantics**: any shape assignment
- Other semantics choose **well-founded assignments**

## DLs in 1990s

descriptive vs. least fixpoint semantics for terminological cycles

## LPs in 1990s

Logic Programs with negation

# Refining the Semantics of Recursive SHACL

- **Supported semantics**: any shape assignment
- Other semantics choose **well-founded assignments**

## DLs in 1990s

descriptive vs. least fixpoint semantics for terminological cycles

## LPs in 1990s

Logic Programs with negation

The validation with recursive shapes is not defined in SHACL and is left to SHACL processor implementations. For example, SHACL processors may support recursion scenarios or produce a failure when they detect recursion.

SHACL Recommendation, §3.4.3

# Giving Semantics to Recursive SHACL

- **Stable model semantics** as in ASP

$$\textit{married} \leftarrow \neg \textit{single} \quad \textit{single} \leftarrow \neg \textit{married}$$

Two stable models:  $\{\textit{married}\}, \{\textit{single}\}$

# Giving Semantics to Recursive SHACL

- **Stable model semantics** as in ASP

$$\textit{married} \leftarrow \neg \textit{single} \quad \textit{single} \leftarrow \neg \textit{married}$$

Two stable models:  $\{\textit{married}\}, \{\textit{single}\}$

- minimal models of the Gelfond-Lifschitz reduct
- alternative level-wise definition for SHACL
- **NP-complete** validation

# Giving Semantics to Recursive SHACL

- **Stable model semantics** as in ASP

$$\text{married} \leftarrow \neg \text{single} \quad \text{single} \leftarrow \neg \text{married}$$

Two stable models:  $\{\text{married}\}, \{\text{single}\}$

- minimal models of the Gelfond-Lifschitz reduct
  - alternative level-wise definition for SHACL
  - **NP-complete** validation
- **Well-founded semantics** are a 3-valued

$$\text{married} \leftarrow \neg \text{single} \quad \text{single} \leftarrow \neg \text{married}$$

Both *married* and *single* undefined

# Giving Semantics to Recursive SHACL

- **Stable model semantics** as in ASP

$$\textit{married} \leftarrow \neg \textit{single} \quad \textit{single} \leftarrow \neg \textit{married}$$

Two stable models:  $\{\textit{married}\}, \{\textit{single}\}$

- minimal models of the Gelfond-Lifschitz reduct
  - alternative level-wise definition for SHACL
  - **NP-complete** validation
- **Well-founded semantics** are a 3-valued

$$\textit{married} \leftarrow \neg \textit{single} \quad \textit{single} \leftarrow \neg \textit{married}$$

Both *married* and *single* undefined

- approximation of stable models
- **Polynomial** validation
- inconsistency tolerant

**Stratified constraints:** with only positive recursion cycles



**Stratified constraints:** with only positive recursion cycles

$$alive \leftarrow \neg dead \quad certified \leftarrow certified$$

**Stratified constraints:** with only positive recursion cycles

$$alive \leftarrow \neg dead \quad certified \leftarrow certified$$

**stable model = well-founded model = Perfect model**

**Stratified constraints:** with only positive recursion cycles

$$alive \leftarrow \neg dead \quad certified \leftarrow certified$$

**stable model = well-founded model = Perfect model**

- $\{alive\}$  is the only model, *certified* is false
- **P-complete** validation

**Stratified constraints:** with only positive recursion cycles

$$alive \leftarrow \neg dead \quad certified \leftarrow certified$$

**stable model = well-founded model = Perfect model**

- $\{alive\}$  is the only model,  $certified$  is false
- **P-complete** validation

In contrast, **supported validation**

- Two models:  $\{alive\}$ ,  $\{alive, certified\}$
- **NP-complete** validation

# Validating SHACL with Ontologies

---

- Open-world semantics: DLs
- Close-world semantics: SHACL

What if we want both?

- Validating constraints in the presence of ontologies

Our data contains:

*hasBird(linda, blu)      hasDog(john, ace)*

- Open-world semantics: DLs
- Close-world semantics: SHACL

What if we want both?

- Validating constraints in the presence of ontologies

Our data contains:

*hasBird(linda, blu)      hasDog(john, ace)*

We want to validate *PetOwnerShape* for John and Linda

*PetOwnerShape*  $\leftarrow \exists hasPet$

- Open-world semantics: DLs
- Close-world semantics: SHACL

What if we want both?

- Validating constraints in the presence of ontologies

Our data contains:

*hasBird(linda, blu)*      *hasDog(john, ace)*

We want to validate *PetOwnerShape* for John and Linda

*PetOwnerShape*  $\leftarrow \exists hasPet$

But we also have a TBox:

*hasBird*  $\sqsubseteq$  *hasPet*      *hasDog*  $\sqsubseteq$  *hasPet*



## Certain Answers?

- SHACL has **negation**
- **Certain answer semantics** too weak!

# Certain Answers?

- SHACL has **negation**
- **Certain answer semantics** too weak!

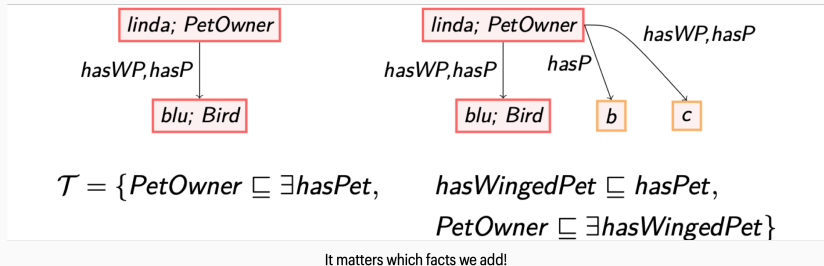
*hasPet(linda, blu)      Bird(blu)*

*otherPetOwner*  $\leftarrow \exists hasPet. \neg Dog$

# Universal models?

For **Horn DLs** we usually rely on the existence of a **universal model** that can be **homomorphically embedded** into **all** models

- aka **chase** in databases



But SHACL expressions are **not preserved under homomorphism!**



Intuitively:

- universal
- does not contain itself as a substructure

Intuitively:

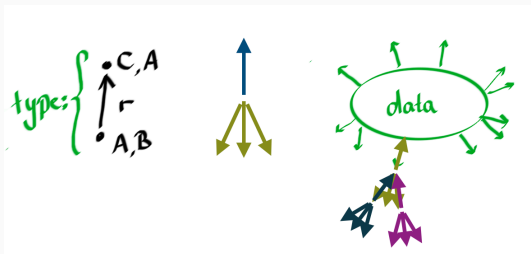
- universal
- does not contain itself as a substructure

Typical constructions check for "coreness" after each chase step

# Austere Model Construction

For Horn DLs like DL-Lite and  $\mathcal{ELHI}$ :

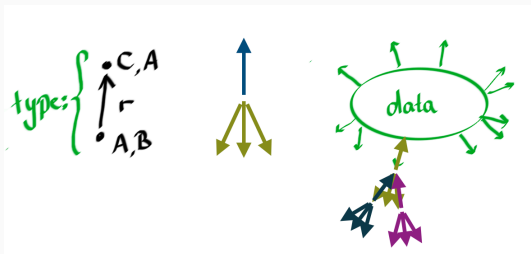
- compute a minimal local successor configuration for each 2-type
- build the model using these successors



# Austere Model Construction

For Horn DLs like DL-Lite and  $\mathcal{ELHI}$ :

- compute a minimal local successor configuration for each 2-type
- build the model using these successors



## Theorem

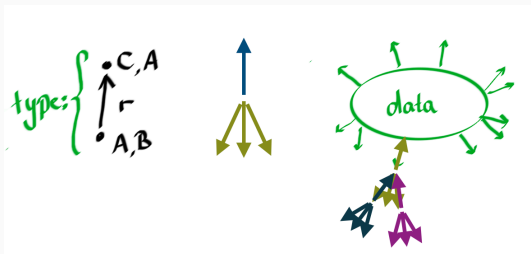
The austere model is a (possibly infinite) universal core



# Austere Model Construction

For Horn DLs like DL-Lite and  $\mathcal{ELHI}$ :

- compute a minimal local successor configuration for each **2-type**
- build the model using these successors



## Theorem

The austere model is a (possibly infinite) universal core

Avoid expensive minimality check after each chase step!

# Validation via Rewriting

Given a TBox  $\mathcal{T}$  and a set of constraints  $\mathcal{C}$ , obtain  $\mathcal{C}$  such that, for every data graph  $G$  and any target  $M$ :

$G$  validates  $(\mathcal{C}_{\mathcal{T}}, M)$   
iff

the austere universal model of  $(\mathcal{T}, G)$  validates  $(\mathcal{C}, M)$

- Calculus to **propagate validated shapes** using the successor configuration
- After saturation, generate new constraints
- Works even with **stratified negation**

## Theorem

*Validation of stratified SHACL constraints in the presence of Horn- $\mathcal{ALCH}$ I ontologies is ExpTime complete in combined complexity and PTime complete in data complexity*

# Validation via Rewriting

Given a TBox  $\mathcal{T}$  and a set of constraints  $\mathcal{C}$ , obtain  $\mathcal{C}$  such that, for every data graph  $G$  and any target  $M$ :

$G$  validates  $(\mathcal{C}_{\mathcal{T}}, M)$   
iff

the austere universal model of  $(\mathcal{T}, G)$  validates  $(\mathcal{C}, M)$

- Calculus to **propagate validated shapes** using the successor configuration
- After saturation, generate new constraints
- Works even with **stratified negation**

## Theorem

*Validation of stratified SHACL constraints in the presence of Horn- $\mathcal{ALCHI}$  ontologies is ExpTime complete in combined complexity and PTime complete in data complexity*

ExpTime hardness holds even for very simple ontologies!

# Validation via Rewriting: an example

From

$$\text{birdShape} \leftarrow \text{Bird} \quad \text{birdOwnerShape} \leftarrow \exists \text{hasPet}.\text{birdShape}$$
$$\text{PetOwner} \sqsubseteq \exists \text{hasPet} \quad \text{hasWingedPet} \sqsubseteq \text{hasPet} \quad \exists \text{hasWingedPet}^- \sqsubseteq \text{Bird}$$

# Validation via Rewriting: an example

From

$$\text{birdShape} \leftarrow \text{Bird} \quad \text{birdOwnerShape} \leftarrow \exists \text{hasPet}.\text{birdShape}$$

$$\text{PetOwner} \sqsubseteq \exists \text{hasPet} \quad \text{hasWingedPet} \sqsubseteq \text{hasPet} \quad \exists \text{hasWingedPet}^- \sqsubseteq \text{Bird}$$

the rewriting obtains new constraints like:

$$\text{birdShape} \leftarrow \exists \text{hasWingedPet}^- \quad \text{birdOwnerShape} \leftarrow \exists \text{hasWingedPet}$$

# What's next?

Many fun questions open:

- Full negation
- Make this work in practice
- Non-Horn DLs
- SHACL optimization
- validation under updates
- ...

Combining Open and Closed-world reasoning still a very exciting area!